MATHEMATICS SPECIALIST UNITS 1&2

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	92	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	6	
10	7	
11	7	
12	8	
13	8	
14	6	
15	8	
16	7	
17	7	
18	7	
19	7	
20	7	
21	7	
S2 Total	92	
S2 Wt (×0.7065)	65%	

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Section Two: Calculator-assumed 65% (92 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) Determine the vector projection of $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ on $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$. (2 marks)

(b) Determine the exact value(s) of t so that the vectors $\begin{pmatrix} t \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3t - 5 \\ 2 \end{pmatrix}$ are

(i) parallel. (2 marks)

(ii) perpendicular. (2 marks)

See next page

Question 10   **(7 marks)**

Two transformation matrices are $T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $S = \begin{bmatrix} 2 & 5 \\ -2 & 5 \end{bmatrix}$.

Triangle ABC has an area of 35 cm², with vertices at $A(-3, -5), B(6, -6)$ and $C(4, 2)$.

(a)   Determine the coordinates of ABC after the triangle has been transformed by matrix T.

(3 marks)

(b)   Use the geometric transformation to explain why the determinant of T is 1.   (1 mark)

(c)   Use the geometric transformation to explain why $T^2 = I$, where I is the 2×2 identity matrix.   (1 mark)

(d)   Determine the area of ABC after the triangle has been transformed by matrix S.   (2 marks)

See next page

Question 11 **(7 marks)**

(a) Five-digit even numbers are to be made using the digits $2, 3, 4, 5, 6$ and 7. Determine how many such numbers exist if the number must exceed $40\,000$ and no digit may be used more than once in a number. (3 marks)

(b) The library in a small guesthouse has 28 different books, of which 19 are fiction and the remainder non-fiction. Determine the number of different ways that a guest can select four books if they want

    (i) the same number of fiction and non-fiction books. (2 marks)

    (ii) at least one non-fiction book. (2 marks)

Question 12 **(8 marks)**

(a) Write the inverse of the true statement 'if a figure is a square then it is a quadrilateral' and use an example or counter-example to briefly discuss the truth of the inverse.

(2 marks)

(b) Points P, Q, R and S lie as shown on a circle with centre O so that PQ is a diameter, $PS = QR$ and $\angle QOR = 36°$.
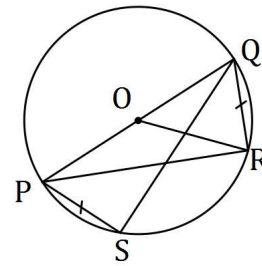
Determine the size of

(i) $\angle QPR$. (1 mark)

(ii) $\angle QPS$. (1 mark)

(iii) $\angle RPS$. (1 mark)

(c) Two chords of a circle, AC and BD, intersect at E so that $AC = 59$ cm, $BE = 33$ cm and $BD = 53$ cm. Determine all possible lengths of AE. (3 marks)

See next page

Question 13 **(8 marks)**

In triangle OAB, P is the midpoint of OB and M is the midpoint of PA. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Show that $\overrightarrow{OM} = \frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}$. (2 marks)

The position vector of A is $\begin{pmatrix} 5 \\ -5 \end{pmatrix}$, position vector of B is $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and O is the origin.

(b) Determine a unit vector $\hat{\mathbf{u}}$ in the same direction as $\overrightarrow{OM}$. (2 marks)

(c) Show that OB is perpendicular to PM. (2 marks)

See next page

(d)   Determine the size of $\angle AOB$.   (2 marks)

Question 14 **(6 marks)**

Consider the following statement:

For two integers a, b if $5a^2 + 2b^2$ is a multiple of 4 then at least one of a, b is even.

(a) Write the contrapositive of the statement. (1 mark)

(b) Prove that the statement is true. (5 marks)

See next page

Question 15   **(8 marks)**

The height of the tide, h cm, of the sea above the mean level at time t hours after midnight one day is given by

$$h(t) = 165 \sin\left(\frac{\pi t}{6}\right) - 52 \cos\left(\frac{\pi t}{6}\right).$$

(a) Express h in the form $a\sin(bt - \theta)$, where $a, b > 0$ and $0 \le \theta \le 2\pi$.   (3 marks)

(b) Determine, to the nearest minute, the time of the first high tide.   (2 marks)

(c) Sketch the graph of the height of the tide on the axes below.   (3 marks)

h

6   12   18   24   *t*

See next page
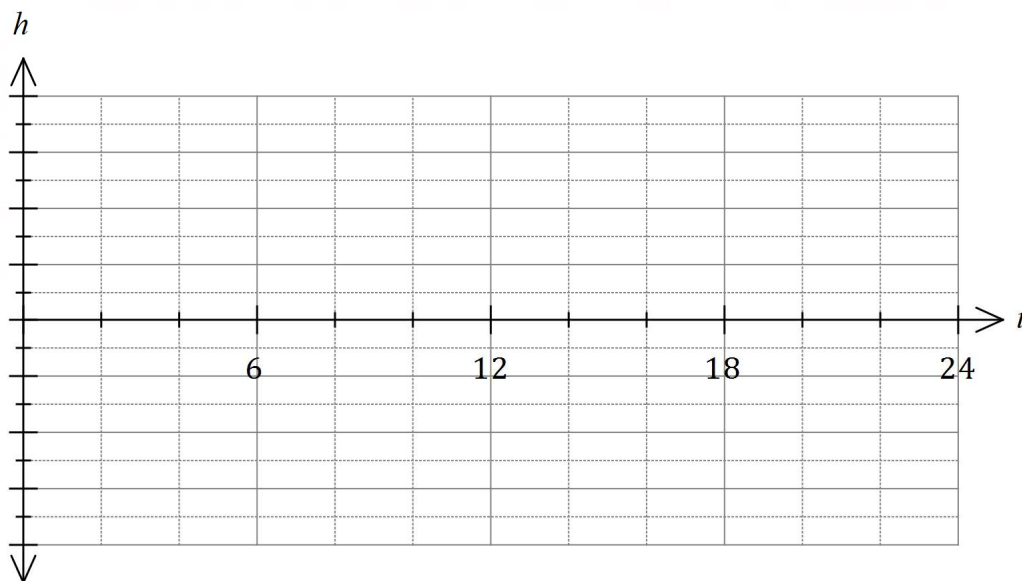
Question 16 **(7 marks)**

(a) In trapezium $OABC$, AC and OB are diagonals, and AB is parallel to OC. Use a vector method to prove that $\overrightarrow{AC} + \overrightarrow{OB} = \overrightarrow{AB} + \overrightarrow{OC}$. (3 marks)

(b) In rectangle $PQRS$, let $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$. Use a vector method to prove that if the diagonals PR and QS are perpendicular, then $PQRS$ is a square. (4 marks)

Question 17 **(7 marks)**

Airport B lies 170 km due east of airport A, and in the region of the airports a wind of 25 km/h is blowing from the northeast.

A small plane, with a cruising speed of 120 km/h, leaves airport A. The pilot, not aware of the wind and intending to fly to airport B, steered the plane on a bearing of 090°.

Assuming that the pilot does not realise their mistake, determine how close the plane will come to airport B if it continues to fly for several hours on the same bearing.

Question 18 **(7 marks)**

Three forces F_1, F_2 and F_3 act on a small body, where $F_1 = 4i - 10j$ N, $F_2 = -8i + 16j$ N and $F_3 = 9i - 15j$ N.

(a) Determine the magnitude of the resultant force and the angle between the resultant force and the vector i. (3 marks)

(b) Two of the forces, F_2 and F_3, can be multiplied by scalars λ and μ respectively so that the three forces are in equilibrium. Determine the value of λ and the value of μ. (4 marks)

See next page

Question 19 **(7 marks)**

(a) 222 people are asked to choose two different letters from those in the word PROBATIVE and write them down in order. Use the pigeonhole principle to prove that at least four people will write the same pair of letters in the same order. (3 marks)

(b) Three character codes, such as PHN, are made using three different letters chosen from the word CHAMPIONED. Determine the proportion of all possible codes that start with a C or end with a D. (4 marks)
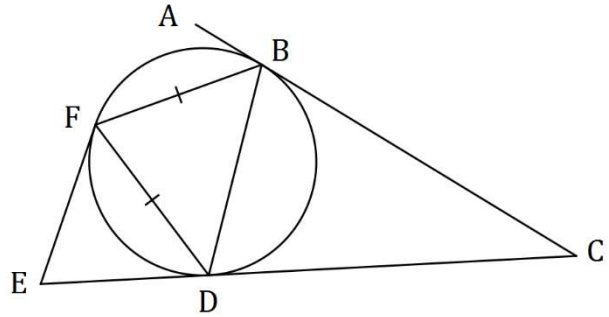
Question 20 **(7 marks)**

The diagram, not drawn to scale, shows vertices B, D and F of an isosceles triangle lying on a circle so that $FD = FB$.

Lines CA and CE are tangential to the circle at B and D respectively.

$CBFE$ is a cyclic quadrilateral.

Let $\angle ACE = \alpha$.

(a) Determine, with reasons, the size of $\angle FEC$ in terms of α. (5 marks)

(b) Hence determine the range of values for the size of $\angle FEC$ in degrees. (2 marks)

See next page

Question 21 **(7 marks)**

Triangle XYZ has vertices $X(6, -2), Y(-5, 5)$ and $Z(2, 9)$.

XYZ is reflected in the line $y = -\dfrac{x}{2}$ to form triangle $X'Y'Z'$.

(a)   Determine the coordinates of Y'. (2 marks)

XYZ is rotated clockwise $45°$ about the origin to form triangle $X''Y''Z''$.

(b)   Determine the coordinates of X''. (2 marks)

(c)   Determine matrix $\boldsymbol{M}$ that will transform $X''Y''Z''$ to $X'Y'Z'$. (3 marks)

Supplementary page

Question number: _____

Supplementary page

Question number: _____